## Chiral effective model with the Polyakov loop

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We discuss how the simultaneous crossovers of deconfinement and chiral restoration can be realized. We propose a dynamical mechanism assuming that the effective potential gives a finite value of the chiral condensate if the Polyakov loop vanishes. Using a simple model, we demonstrate that our idea works well for small quark mass, though there should be further constraints to reach the perfect locking of two phenomena.

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Introduction Chiral symmetry plays an important role in effective model approaches to Quantum Chromodynamics (QCD). The hadronic properties at low energy have been successfully described by chiral effective models such as the linear sigma model [1], the Nambu–Jona-Lasinio (NJL) model [2, 3], the chiral random matrix model [4], chiral perturbation theory [5] and so on. The nature of the chiral phase transition at finite temperature can be classified by chiral symmetry according to the universality argument [6] and investigated in these effective models [7]. In particular, in the massless two-flavor case, we can expect a chiral phase transition of second-order that belongs to the same universality class as the 3d O(4) spin model. Then, we can anticipate what could occur in the real world with finite but sufficiently small (up and down) quark masses.

The deconfinement phase transition is rather obscure and veiled because it is only well-defined in the heavy quark limit, which is too far from the real world. In the heavy quark limit, i.e., in the absence of dynamical quarks, the Polyakov loop serves as an order parameter for deconfinement and the phase transition is characterized by the spontaneous breaking of center symmetry [8]. In the presence of dynamical quarks the center symmetry is explicitly broken and no order parameter or criterion has been established for the deconfinement transition [9]. As mentioned above, on the other hand, the chiral phase transition at high temperature or baryon density has been well-understood by means of effective models. Those model studies based on chiral symmetry, however, lack any dynamics coming from the Polyakov loop, except for some efforts to clarify the interplay between chiral dynamics and the Polyakov loop [10, 11, 12, 13, 14].

It is often argued that the mixing between the Polyakov loop, L, and the chiral order parameter (chiral condensate),  $\chi$ , can account for the lattice QCD observation that deconfinement and chiral restoration occur at the same pseudocritical temperature [15]. The mixing argument is, however, not sufficient to give a satisfactory explanation on the lattice QCD data. In the  $(T, m_q)$  plane  $(m_q)$  being the current quark mass), as discussed in [16], there appear two terminal points of the first-order phase boundary, namely, the critical end-points (CEPs). One is the chiral CEP denoted by  $(T_C, m_q^C)$  and the other is the deconfinement CEP denoted by  $(T_D, m_q^D)$ . For example, in the two-flavor three-color case, it is known that  $(T_C \sim 170 \text{ MeV}, m_q^C = 0)$  and  $(T_D \sim 270 \text{ MeV}, m_q^D \sim 1 \text{ GeV})$  [13, 17]. It should be noted that the CEP is a true second-order critical point with a divergent susceptibility. Mixing means that L and  $\chi$  should share the same singularity in their susceptibilities near the CEP; for  $m_q = m_q^C$  (and  $m_q^D$ ), the susceptibilities of L and  $\chi$  both diverge at  $T = T_C$  (and  $T_D$  respectively). The susceptibility peak is smeared as  $m_q$  leaves from the CEP. The important point is that, with  $m_q$  fixed at a certain value near the CEP, a smeared bump originating from the other CEP may be observed separately as well as a sharp peak from the closer CEP. Thus the mixing argument cannot exclude a double-peak structure. For  $m_q \simeq m_q^C$  for example, the Polyakov loop susceptibility can have a sharp peak around  $T_C$  (coming from the mixing with the diverging chiral susceptibility) as well as a broad bump around  $T_D$  (coming from a remnant of deconfinement). We should be cautious about the mixing argument to understand the lattice QCD data in which no double-peak structure has been seen for any  $m_q$  [15, 16].

In fact, the *locking* between the two crossover phenomena depends on the detailed properties of the interaction. The purpose of this letter is to propose a simple mechanism to exclude the undesirable double-peak structure and to complement the shortcomings of the mixing argument.

Idea To make our idea clear in general setting, let us suppose that we have a full effective potential, i.e.,  $V_{\text{eff}}[L,\chi;m_q]$ . In principle, the behavior of L and  $\chi$  should be completely determined by  $V_{\text{eff}}[L,\chi;m_q]$ . Thus the question arises; what property of  $V_{\text{eff}}[L,\chi;m_q]$  can give rise to the simultaneous crossovers? Our idea is as follows.

First of all, an important property follows from the theoretical arguments given by Casher [18] and 't Hooft [19].

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According to their arguments, the confined phase must have a non-vanishing chiral condensate, which suggests that the chiral phase transition should occur at higher temperature than deconfinement. This means that  $V_{\text{eff}}[L=0,\chi;m_q=0]$ leads to  $\chi \neq 0$  at any temperature if L=0 is imposed by hand (or approximately chosen as a minimum of the effective potential). This property has not been proven in QCD (see also [20]) but is realized in the strong coupling analysis [13] and assumed here.

Next, because L has turned out to behave approximately as an order parameter in lattice simulations [15], we can expect that L is almost zero below the deconfinement crossover temperature,  $T_{\rm d}$ , regardless of dynamical quarks. Then, together with the above property,  $\chi$  must have a non-vanishing value below  $T_{\rm d}$  even for  $m_q \sim 0$  (i.e., spontaneous chiral symmetry breaking). Thus the chiral restoration temperature,  $T_{\chi}$ , is greater than or equal to  $T_{\rm d}$ .

In contrast, the critical temperatures at the CEPs are  $T_{\rm D} \simeq 270~{\rm MeV}$  (for  $m_q = \infty$ ) >  $T_{\rm C} \simeq 170~{\rm MeV}$  (for  $m_q = 0$ ),

which implies  $T_{\rm d} \geq T_{\chi}$  for an intermediate value of  $m_q$ , unless chiral or center symmetry is overwhelmingly broken. Our idea is that  $T_{\rm d} = T_{\chi}$  is likely to be realized by  $T_{\chi} \geq T_{\rm d}$  from the properties of the effective potential and  $T_{\rm d} \geq T_{\chi}$  from, so to speak, the boundary condition. Chiral symmetry is broken by  $m_q \neq 0$ , while the center symmetry breaking is suppressed by the constituent quark mass even for small  $m_q$ . Hence, our idea is expected to work especially for  $m_q \sim 0$ . This mechanism can complement the mixing argument and lead to a robust single-peak structure in the susceptibilities.

Effective Model For the purpose of demonstrating our idea, we propose a simple chiral effective model with Polyakov loop dynamics. If the Polyakov gauge ( $A_4$  is static and diagonal) is employed, the Polyakov loop (or strictly speaking its phase) appears in the quark action as an imaginary quark chemical potential [22, 23]. Thus we can uniquely determine the coupling between the Polyakov loop and quark excitations. We shall take this advantage by adopting the NJL model which is given in terms of quark degrees of freedom.

The conventional Lagrangian density of the NJL model is

$$\mathcal{L}_{\text{NJL}} = \bar{q} \left( i \gamma^{\mu} \partial_{\mu} - m_q \right) q + \frac{G}{2} \left\{ (\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2 \right\}, \tag{1}$$

where  $m_q = 5.5 \,\mathrm{MeV}$  and  $G = 2 \times 5.496 \,\mathrm{GeV}^{-2}$ . The momentum integration is regulated by the cut-off  $\Lambda = 631 \,\mathrm{MeV}$ . These model parameters are chosen as to reproduce the pion mass and decay constant at zero temperature [3]. In the mean field approximation the thermodynamic potential is given by

$$\Omega_{\rm NJL}/V = \frac{1}{2G}(M - m_q)^2 - 2N_{\rm c}N_{\rm f} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \Big\{ E_p + T \ln \left[ 1 + \mathrm{e}^{-(E_p - \mu)/T} \right] + T \ln \left[ 1 + \mathrm{e}^{-(E_p + \mu)/T} \right] \Big\}.$$
 (2)

V is the spatial volume,  $N_{\rm f}$  is the flavor number fixed as  $N_{\rm f}=2$  throughout this letter, and  $\mu$  is the quark chemical potential. We neglect any  $\mu$  dependence in G as usual [3]. The energy of quasi-quarks is given by  $E_p = \sqrt{p^2 + M^2}$ with the constituent quark mass  $M = m_q - G\langle \bar{q}q \rangle$ . The cut-off is imposed only on the first term in the curly brackets (zero-point energy) in the present analysis. The finite temperature contribution has a natural cut-off in itself specified by the temperature. Identifying the imaginary quark chemical potential with the Polyakov loop, we can define our model by the following thermodynamic potential,

$$\Omega/V = V_{\text{glue}}[L] + \frac{1}{2G}(M - m_q)^2 
- 2N_{\text{c}}N_{\text{f}} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \Big\{ E_p + T \frac{1}{N_c} \text{Tr}_{\text{c}} \ln \Big[ 1 + L \mathrm{e}^{-(E_p - \mu)/T} \Big] + T \frac{1}{N_c} \text{Tr}_{\text{c}} \ln \Big[ 1 + L^{\dagger} \mathrm{e}^{-(E_p + \mu)/T} \Big] \Big\},$$
(3)

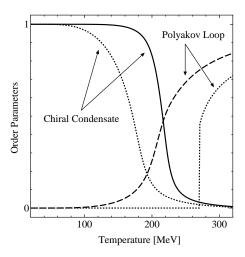
where the Polyakov loop is an  $SU(N_c)$  matrix in color space explicitly given by

$$L(\vec{x}) = \mathcal{T} \exp\left[-i \int_0^\beta dx_4 A_4(x_4, \vec{x})\right]. \tag{4}$$

This coupling between the Polyakov loop and the chiral condensate can be derived also in the strong coupling approach [11, 13]. The generalization to aQCD (QCD with dynamical quarks in the adjoint representation in color space) is readily available if one replaces the Polyakov loop by the Polyakov loop in the adjoint representation (c.f. [13]). In this letter we will focus only on the case in the fundamental representation.

Since the four-quark coupling constant, G, contains the information on gluons, G should depend on L. We simply neglect this possible L dependence. Nevertheless, this L dependence makes no qualitative difference because Gincorporates all gluons and would not be much affected by L only, where L is essentially the temporal component of gluons. This approximation is acceptable in the same level as neglecting the possible  $\mu$  dependence in G.

It is worth noting that the NJL model with both quarks and the Polyakov loop is not incompatible with confinement at low temperature. To make this clear, let us assume that confinement corresponds to the condition, L=0, though



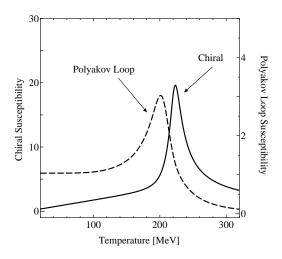


FIG. 1: The left figure is the behavior of the traced Polyakov loop l and the chiral condensate  $\chi/\chi_0$  (normalized by the value at T=0) at  $\mu=0$ . The dotted curves represent  $\chi/\chi_0$  calculated from  $\Omega_{\rm NJL}$  and l from  $V_{\rm glue}$ . The right figure is the susceptibility.

this is not precise due to the center symmetry breaking. The Taylor expansion of the logarithmic terms generates a series in powers of L whose power corresponds to the quark excitation number.  $L, L^2, L^4, \ldots$  terms have nontrivial triality and break the center symmetry, while  $L^3, L^\dagger L = 1, \ldots$  terms have zero triality (color-singlet part) corresponding to baryons, mesons, etc. In a more elaborated approximation scheme used in [9, 13], the group integration of L with a vanishing mean-field, L = 0, singles out only the terms with zero triality. [Our notation is sloppy as long as no confusion arises. L is used for both the Polyakov loop matrix and its traced expectation value.] Roughly speaking, the limit of L = 0 allows only excitations with zero triality such as baryons and mesons. In the present treatment the same argument holds in part (see (6)). Although our model (3) is described in terms of colored quarks, the thermal excitation must consist of colorless composites as long as L is small. Once L gets larger at higher temperature, our model would reduce into the standard NJL model allowing thermal quark excitations.

The remaining part,  $V_{\text{glue}}[L]$ , is the effective potential only in terms of the Polyakov loop. Here we shall adopt a simple choice for  $V_{\text{glue}}[L]$  at the sacrifice of quantitative accuracy; we employ the leading order result of the strong coupling expansion that is simple and yet reasonable qualitatively as compared with the lattice results [24]. This potential has only one parameter a (the lattice spacing). For  $N_c = 3$  we can write it as

$$V_{\text{glue}}[L] \cdot a^3 / T = -2(d-1) e^{-\sigma a / T} \left| \text{Tr}_{c} L \right|^2 - \ln \left[ -\left| \text{Tr}_{c} L \right|^4 + 8 \text{Re} \left( \text{Tr}_{c} L \right)^3 - 18 \left| \text{Tr}_{c} L \right|^2 + 27 \right]$$
 (5)

with the string tension  $\sigma = (425 \,\mathrm{MeV})^2$ . The first term comes from the kinetic part and the second term is just the logarithm of the Haar measure associated with the SU(3) group integration.  $V_{\mathrm{glue}}[L]$  leads to a first order phase transition with the critical coupling  $2(d-1)\mathrm{e}^{-\sigma a/T_{\mathrm{d}}} = 0.5153$ . We can fix the deconfinement transition temperature as the empirical value  $T_{\mathrm{d}} = 270 \,\mathrm{MeV}$  by choosing  $a^{-1} = 272 \,\mathrm{MeV}$ .

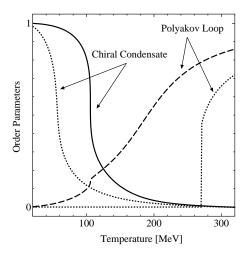
Apparently, the present model has two cut-offs, i.e.,  $\Lambda$  and  $a^{-1}$ . This means that the model has two independent scales for chiral symmetry breaking and confinement, or for mesons and the Polyakov loop. Since QCD has only one scale,  $\Lambda$  and  $a^{-1}$  should be related to each other in principle. Here we simply fix them as model parameters since we are working in the effective model to abstract the essence for each dynamics [21].

Numerical Results Before we refer numerical calculations in the  $N_{\rm c}=3$  case, let us introduce an ansatz to simplify the analysis. In the Polyakov gauge we can parametrize the SU(3) Polyakov loop matrix as  $L={\rm diag}({\rm e}^{{\rm i}\phi},{\rm e}^{{\rm i}\phi'},{\rm e}^{-{\rm i}(\phi+\phi')})$ . The perturbative vacuum has  $\phi=\phi'=0$  and we can choose the confining vacuum at  $\phi=2\pi/3,\,\phi'=0$  [22]. Thus we shall fix  $\phi'=0$  from the beginning for simplicity. Once this ansatz is accepted, we can rewrite the potential (3) only in terms of the traced Polyakov loop, i.e.  $l=({\rm Tr}_{\rm c}L)/N_{\rm c}=(1+2\cos\phi)/3$ . After straightforward calculations we can reach the expression;

$$\operatorname{Tr}_{c} \ln \left[ 1 + L e^{-(E_{p} - \mu)/T} \right] + \operatorname{Tr}_{c} \ln \left[ 1 + L^{\dagger} e^{-(E_{p} + \mu)/T} \right]$$

$$= \ln \left[ 1 + (3l - 1) e^{-(E_{p} - \mu)/T} + e^{-2(E_{p} - \mu)/T} \right] + \ln \left[ 1 + (3l - 1) e^{-(E_{p} + \mu)/T} + e^{-2(E_{p} + \mu)/T} \right]$$

$$+ \ln \left[ 1 + e^{-(E_{p} - \mu)/T} \right] \ln \left[ 1 + e^{-(E_{p} + \mu)/T} \right]. \tag{6}$$



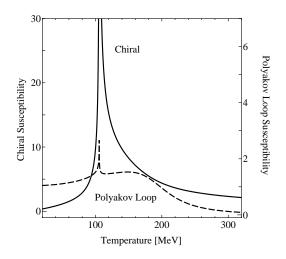


FIG. 2: The order parameter and susceptibility around the chiral critical end-point with  $\mu_{\mathsf{E}} = 321\,\mathrm{MeV}$ .

When l=1, the model is reduced into the standard two-flavor NJL model having the chiral phase transition at  $T_\chi=175\,\mathrm{MeV}$  in the chiral limit. If l is forced to be zero by hand, the temperature effect is so suppressed that spontaneous chiral symmetry breaking can sustain until  $T_\chi\simeq 520\,\mathrm{MeV}$ , that is much higher than  $T_\mathrm{D}$ . Therefore the chiral phase transition cannot occur until the Polyakov loop jumps from nearly zero to a certain finite value. Our model (3) satisfies the essential assumption in our idea,  $T_\chi\geq T_\mathrm{d}$ , for  $m_q\sim 0$ .

Figure 1 (left) shows the resulting behavior of order parameters as functions of temperature at  $\mu=0$ . To see the simultaneous crossovers clearly the chiral condensate  $\chi=\langle\bar{q}q\rangle$  is normalized by the value at T=0 (denoted by  $\chi_0$ ). The results from  $\Omega$  are represented by the solid (for  $\chi/\chi_0$ ) and dashed (for l) curves. The dotted curves are the results from  $\Omega_{\rm NJL}$  and  $V_{\rm glue}$  without any interaction between  $\chi$  and l for reference. It should be noted that the mixing interaction between  $\chi$  and l vanishes at zero temperature in this model. Consequently the normalization  $\chi_0$  is identical for both the chiral condensates from  $\Omega$  and  $\Omega_{\rm NJL}$ .

In the presence of dynamical quarks, as seen from the figure, the Polyakov loop shows a crossover around the pseudo-critical temperature  $T_{\rm c} \simeq 200\,{\rm MeV}$ . At the same time the chiral condensate is affected by the Polyakov loop such that it tends to be almost constant as long as  $T < T_{\rm c}$ . The pseudo-critical temperature can be read from the peak position of each susceptibility. Here we shall define the dimensionless susceptibility of the chiral order parameter and the Polyakov loop by using the curvature inferred from the potential (3). First, we define the dimensionless curvature matrix C by

$$C_{qq} = \frac{\Lambda^2 \partial^2}{\partial M^2} \frac{\beta \Omega}{\Lambda^3 V}, \quad C_{ll} = \frac{\partial^2}{\partial l^2} \frac{\beta \Omega}{\Lambda^3 V}, \quad C_{ql} = C_{lq} = \frac{\Lambda \partial^2}{\partial M \partial l} \frac{\beta \Omega}{\Lambda^3 V}.$$
 (7)

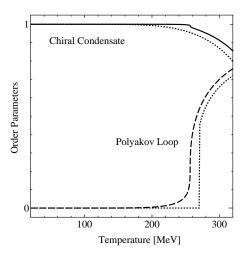
Roughly speaking,  $C_{qq}^{-1}$  ( $C_{ll}^{-1}$ ) corresponds to the chiral (Polyakov loop) fluctuation and  $C_{ql}$  is the interaction vertex of a quark and the Polyakov loop. Then, the susceptibility is given by the inverse of C;

$$\chi_q = (C^{-1})_{qq} = \frac{C_{ll}}{C_{qq}C_{ll} - C_{ql}^2}, \quad \chi_l = (C^{-1})_{ll} = \frac{C_{qq}}{C_{qq}C_{ll} - C_{ql}^2}.$$
 (8)

The physical meaning of the above equations is transparent if we notice that the fraction can be expanded as  $\chi_q = C_{qq}^{-1} + C_{qq}^{-1}C_{ql}C_{ll}^{-1}C_{lq}C_{qq}^{-1} + \cdots$ , that is the sum over mixing contributions. The mixing pattern is similar to the relation between the chiral susceptibility and the baryon number susceptibility [25, 26]. As shown in Fig. 1 (right), our idea works pretty well so that the peak of the Polyakov loop susceptibility can be found just near the peak of the chiral susceptibility, though perfect coincidence is not reached.

When the quark chemical potential,  $\mu$ , becomes larger, the Polyakov loop shows a crossover with smoother slope because the density effect itself breaks the center symmetry explicitly. It is widely accepted that the chiral phase transition becomes of first-order at large  $\mu$ . In the  $(\mu, T)$  plane, therefore, we can expect another CEP [27, 28]. Actually we found the CEP at  $(\mu_{\rm E}=321\,{\rm MeV},\,T_{\rm E}=106\,{\rm MeV})$ , which is close to the value originally obtained in [27]. The order parameter and the susceptibility around this CEP are shown in Fig. 2.

At  $\mu=0$ , as discussed in Introduction, the deconfinement CEP is important as well as the chiral CEP. [In the present two-flavor case the chiral CEP trivially lies at  $m_q=0$ .] We found the deconfinement CEP at  $(m_q^{\sf D}=788\,{\rm MeV},\,T_{\sf D}=257\,{\rm MeV})$  in our model. This value is consistent with the lattice observation in the two-flavor case [17] and also



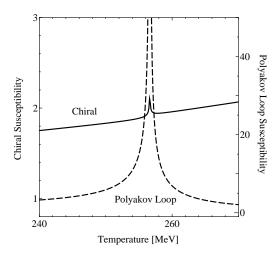


FIG. 3: The order parameter and susceptibility around the deconfinement critical end-point with  $m_p^0 = 788 \,\mathrm{MeV}$  and  $\mu = 0 \,\mathrm{MeV}$ .

in good agreement with the Gocksch-Ogilvie model [13]. The order parameter and the susceptibility around the deconfinement CEP are shown in Fig. 3. Since the deconfinement transition is of second-order, the Polyakov loop susceptibility diverges at  $T = T_D$ . In Figs. 2 and 3 we can see that the chiral susceptibility and the Polyakov loop susceptibility both have a singularity due to the mixing.

**Discussion** We shall briefly summarize characteristic features of our model defined by Eqs. (3) and (6) below.

- 1. The coupling between the Polyakov loop and the chiral condensate is determined uniquely and is consistent with the conventional form; in the leading order of the hopping parameter ( $\kappa$ ) expansion on the lattice, the coupling term takes the form of  $(2\kappa)^{N_{\tau}}l \sim l e^{-M/T}$  [10, 29] (see the coupling term of (3)).
- 2. Because the coupling term is  $\sim l \, \mathrm{e}^{-M/T}$ , it goes to zero as  $T \to 0$ . Actually the zero temperature system can be different from the system at infinitesimally small but finite temperature. At zero temperature, the canonical description with the quark triality fixed at zero is likely to be valid [9].
- 3. Contrary to naive expectation, the Polyakov loop behavior hardly reflects the singularity associated with the chiral phase transition of second-order. [The gross feature of Fig. 1 is hardly changed when  $m_q = 0$  except that  $\chi_q$ 's peak becomes divergent.] This is because the coupling  $C_{ql}$  (amplitude between the Polyakov loop and the chiral condensate) is proportional to the constituent quark mass, M, and vanishes at the chiral phase transition of second-order. Nevertheless, our idea leads to the almost simultaneous crossovers in a robust way.
- 4. Figure 2 is an interesting prediction from our model at finite density. Our idea would not necessarily give the simultaneous crossovers at high density where  $T_{\mathsf{E}}$  is too lower than  $T_{\mathsf{D}}$  and the Polyakov loop has a long tail. This double-peak structure with a sharp peak and a broad bump would be a realistic possibility to be seen in the future lattice simulation at high density.
- 5. As discussed in the section "Idea", our idea would not work for large  $m_q$  because of explicit symmetry breaking. In general the pseudo-critical temperature  $T_\chi$  gets larger with increasing  $m_q$ . Actually the chiral susceptibility peak calculated in the standard NJL model yields  $T_\chi = 270~{\rm MeV}(\sim T_{\rm D})$  at  $m_q = 167~{\rm MeV}$ . For  $m_q > 167~{\rm MeV}$ , our model leads to a double-peak structure with  $T_{\rm d} < T_\chi$ , that is not prohibited by our dynamical mechanism. The chiral susceptibility in Fig. 3 has a second broad bump at much higher temperature than shown in the figure. Although the present model embodies our idea and describes the simultaneous crossovers for small  $m_q$ , there must be some further constraints, in particular to impose  $T_{\rm d} \ge T_\chi$ . Such a condition would realize a perfect locking for small  $m_q$  and cure the failure for larger  $m_q$ .

**Summary** We proposed an idea to realize the simultaneous crossovers of deconfinement and chiral restoration, which turned out to work well for small quark mass. We demonstrated the idea by using a chiral effective model with the Polyakov loop. The model study yields the chiral CEP at ( $\mu_E = 321 \,\mathrm{MeV}$ ,  $T_E = 106 \,\mathrm{MeV}$ ) and the deconfinement CEP at ( $m_q^D = 788 \,\mathrm{MeV}$ ,  $T_D = 257 \,\mathrm{MeV}$ ). The Polyakov loop susceptibility and the chiral susceptibility both diverge at the CEP due to the mixing effect.

Since the thermodynamic potential in our model is a function of  $M^2$  (M being the constituent quark mass), the mixing effect  $\propto M$  is small at  $T = T_{\mathsf{C}}$ . Then our idea plays an essential role to attract one crossover to the other.

Also we presented a prediction from our model at finite baryon density. At sufficiently high density we can expect that the Polyakov loop has a double-peak structure with a sharp peak from the mixing and a broad bump from a remnant of deconfinement.

The present model lacks some mechanism necessary to sustain the locking for  $m_q > 167$  MeV. In other words, this result suggests that some dynamical mechanism is further needed in order that the chiral and deconfinement CEPs are connected by a single crossover line [16]. The scenario of [16] requires something beyond the mixing argument and the present idea to lock two phenomena. Although this is still an open question, we believe that our model can contain correct physics at least for small  $m_q$  and can be a simple starting point to examine the underlying relation between deconfinement and chiral restoration not only at finite temperature but at finite baryon density also.

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